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Pricing Options with Modified Time-Fractional BS. Equation and with Samudu Transform

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ABSTRACT

Background: The maximum payoff is always the wish of a successful business person, and it is possible only when the risk is minimized and they maximize their gain. Even now, it is a big challenge for financial managers of different investors to predict the minimized risk values in a random environment. Specifically, the stock exchange is a good example of option pricing. Option pricing theory is very important to companies and companies because it estimates the fair value of options that will be used to design various future pricing strategies.

Purpose of the Study: The purpose of the study is to predict risk-free prices for two stocks by using modified Black-Scholes partial differential equations in the fractional time sequence for two stocks that have been worked on very little or not at all before.

Finding: The present study finds out that the Samudu Transformation method have a significant role to get the better solution of two dimensional time fractional modified Black-Scholes partial differential equation to make better predication of company shares selling and purchasing.

Value/Implications: This paper provides a new solution for research scholars, bankers, practitioners, and government policy-making departments on how the risk free rates for two-dimensional stocks may be obtained. Finance managers play a critical role in the advancement of the country.

Keywords: Options, pricing, call option, put options, Black Sholes equation, risk free rates, series solution, Samudu Transformation.

Article info.
Received: June 8, 2021
Accepted: December 21, 2021
***Funding Source:** Nil*
***Conflict of Interest:** Nil*

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***Cite this article:** Zakaria K, Hafeez S, Nadeem S, Ishtiaq A, Huma S. (2021). Pricing Options with Modified Time-Fractional BS. Equation and with Samudu Transform. RADS Journal of Business Management, 3(2): 201-215.*

1. RESEARCH BACKGROUND

There are many researchers who gave different methods to evaluate solutions of the BS-Model for only one-dimensional PDE. These are analytical and numerical methods that demonstrate the solutions in the form of values of pricing options. Such analytical methods are the Laplace Adomain Decomposition Method, Natural Transformation method, Homotopy perturbation method, Adomain decomposition method, projected

Differential Transformation method, and Alzaki Transformation method, etc., while numerical methods are the Finite difference method, Finite element method, and Haar Wavelet method. But in this study, we are the first one to use the application of the SAMUDU TRANSFORMS METHOD to investigate the analytical solution of the 2-Dimensional, Time Fractional-ordered BS-Model. It consists of two different assets in Liouville-Caputo Fractional derivative form for the options pricing. The Samudu Transform provides the value of pricing options in the form of explicit solutions in convergent infinite series. This study consists of a new methodology, new concept, and new vision to evaluate the option pricing of the 2-Dimensional, time-fractional-ordered Black-Sholes Model for two stocks.

Statement of the Problem

The primary goal of this thesis is to investigate the use of new well-known and practical integral transform methods, such as the Sumudu Transform, to evaluate option pricing of the Fractional Order Black Sholes Model for stock(s).

2. LITERATURE REVIEW

According to the research of Guillaume (2019), option pricing is regarded as an effective solution to various pricing problems in global financial markets. The scope of option pricing is very important for local and international commercial and economic activities because it provides a method to predict asset prices in any financial market.

Boer (2019) believes that every entrepreneur is seeking to maximise the utility and benefits of doing business. However, this can only be achieved by reducing risks and increasing business activities. For example, the stock market can be viewed as a business platform with highly unpredictable and uncertain stock prices. However, using the Black-Scholes PDE model can help achieve risk-free pricing in these regions.

Guillaume (2019) believes that the fractional financial business model is expressed in the random PDE in order to manage changes, achieve higher accuracy, achieve greater tolerance, and learn the nature of financial markets. The Black-Scholes model is widely used to predict option prices. According to this model, instability is not eliminated.

According to the research of Moutsinga (2018), the valuation of stocks and options is very important in the financial markets. Over time, the use of derivatives has increased because they provide useful solutions to various mathematical models that help solve complex financial problems. and predict the behaviour of financial markets. The score calculation serves as a link between the model and the financing of the business solution.

Esekon (2016) believes that option pricing usually uses some models, among which the Black-Scholes mathematical model is very simple and robust. various exchanges, financial markets, and complex production issues. However, it is also very important to know that this model can be used as a reference for future finances, because it is the first model that helps predict options and implied volatility.

According to Rubinstein (1994), plenitude kurtosis is related to determining insecurity for the S&P 500. Shimko (1993) concluded in his ponders that induced conveyances of S&P 500 records are conflictingly leptokurtic and inclined. By Jackwerth (1996), it appears that lognormal apportionments are connected within the scattering of the S&P 500. Before 1987, it deteriorated to see things like negative skewness and leptokurtosis.

Geske and Roll (1984) examined that there's an unsteady inclination in both the cash and outright money choices at a special time. Geske and Roll (1984) concluded that cash and time inclination can be distinguished with inappropriate constraint conditions, while the issue of unusualness slant may result from botches in performing estimation.

Galai (1977) believes that the BS model reveals that the speculative moment of instability has to be eased. Galai's (1977) findings are comparable to Geske's (1977) findings (1979). According to MacBeth (1980), the unpredictability of the hidden means that when there is an increase in stock value, the odds of danger become less. Beckers (1980) used Black-Scholes suspicion on the S&P 500 record alternative.

In this article, the method of Samudu transformation is used for the demonstration of a systematic solution of the two-way time-division BS model, which is composed of two unique assets. This conversion gives the value of the put option as an explicit solution in the form of a convergence series. In the Laplace perturbation method, the Laplace transform method is applied in the first stage, and then the homotopy method is applied. It is feasible, but in the Samudu transformation method, a solution can be found without much work. The solution that is taken in the Laplace homotopy is similar to this method. The next part will introduce the methods to solve the two assets of the BS financial model.

3. METHODOLOGY

Let us take the following PDE:

$$D^{\alpha}\,\psi\,(m,n,o) + \lfloor\psi(m,n,o) + N\psi(m,n,o) = Z ----(1)$$

Where $j-1 < \propto \leq j$; $\quad j \in N$

which goes to:

$$\psi(m,n,0) = \psi_0\,(m,n)$$

Where $L =$ LDO (It stands for Linear Differential operator

$N =$ NLDO (It stands for Non -Linear Differential operator)

here, because of equation 1, the transformation for $\psi(m,n,o)$ can be find out as:

$$S[D^{\alpha}\,\psi\,(m,n,o)] = s[\,\psi(m,n,o) - \lfloor\psi(m,n,o) - N\psi(m,n,o) + Z -------- (2)$$

$$S[D^{\alpha}\,\psi\,(m,n,o)] = b^{-\propto}s[\psi(m,n,o) - \sum_{k=0}^{j-1} b^{-\propto+k}\,\frac{\partial^k}{\partial t^k}\,\psi(m,n,0)$$

If $\propto < 1$, setting $j = 1$

$$S[D^{\alpha}\,\psi\,(m,n,o)] = b^{-\propto}s\,\psi(m,n,o) - b^{-\propto}\,\psi(m,n,0)$$

$$b^{-\propto}s\,\psi(m,n,o) - b^{-\propto}\,\psi(m,n,0) = s\,[Z - \lfloor\,\psi(m,n,o) - N\psi(m,n,o)$$

$$s\,\psi(m,n,o) = \psi(m,n,0) + b^{-\propto}s\,[z - \lfloor\,\psi(m,n,o) - N\psi(m,n,o)$$

Here we need to implement Inverse Sumudu Transform:

$$s\,s^{-1}\psi(m,n,o) = s^{-1}\,\psi(m,n,0) + s^{-1}\{b^{\propto}s\,[Z - \lfloor\,\psi(m,n,o) - N\psi(m,n,o)]\}$$

Now we need to call Inverse property of S.T(Samudu Transform)

$$I^{\propto}\,[h(m,n,o)] = s^{-1}[b^{\propto}s(h(m,n,o)]$$

here is the need of Integral Property of S.T implementation on third equation:

$$\psi(m,n,o) = \psi_0(m,n) + I^{\propto}\,[Z - \lfloor\,\psi(m,n,o) - N\psi(m,n,o) ------ (4)$$

The solution of PDE in form of infinite convergent series is expressed by samudu transform as mentioned below:

$$\psi(m,n,o) = \psi_0(m,n) + \sum_{j=1}^{\infty} \frac{h_j(m,n)o^{j\propto}}{\Gamma(1+j\propto)}$$

Where

$\psi_0(m,n) = h_0(m,n) = h_0$

$h_1 = Z - \lfloor(h_0) - N(h_0)]$

$h_2 = Z - \lfloor(h_1) - N(h_1)]$

$h_3 = Z - \lfloor(h_2) - N(h_2)]$

$\vdots \qquad \vdots$

$$h_j = Z - \lfloor h_{(j)} - N[h_{(j)}]$$

Since the non-linear PDE is also a two stocks fractional order black sholes model, can also be solved by sumudu integral transform. The famous BS equation is written below.

$$\frac{\partial^{\propto} C}{\partial t^{\propto}} + \frac{\sigma_1^2}{2} S_1^2 \frac{\partial^2 C}{\partial S_1^2} + \frac{\sigma_2^2}{2} S_2^2 \frac{\partial^2 C}{\partial S_2^2} + rS_1 \frac{\partial C}{\partial S_1} + rS_1 \frac{\partial C}{\partial S_1} + rS_2 \frac{\partial C}{\partial S_2} + p\, S_1 S_1 \sigma_1 \sigma_2 \frac{\partial^2 C}{\partial S_1 \partial S_2} - rC = 0$$

Investor's pay-off relation:

$$c(S_1, S_{2\,,}o) = \max\,(w_1\; S_1 + wS_2 - \;k\,,0) \qquad \text{(European)}$$

$$\mathrm{P}(S_1, S_{2\,,}o) = \max\,(w_1\; S_1 + wS_2 - \;k\,,0) \qquad \text{(American)}$$

Where

C= European

P = American

$S_1 =$ asset 1 cost share

$S_2 =$ asset 2 cost share

p= correlation coefficient among cost of shares of asset 1 and asset 2

σ_1= cost volatility 1

σ_2= cost volatility 2

K = Strike cost

w_1= asset 1 investment properties

w_2= asset 2 investment properties

We can consider the substitution method to simplify the forth equation

$$b = lnS_1 - \left(r - \frac{1}{2}\sigma_1^2\right)o$$

$$d = lnS_2 - \left(r - \frac{1}{2}\sigma_2^2\right)o$$

$\frac{\partial C}{\partial S_1} = \frac{\partial C}{\partial d}\frac{\partial b}{\partial S_1}$ $\qquad$ $\frac{\partial C}{\partial S_2} = \frac{\partial c}{\partial d}\frac{\partial b}{\partial S_2}$c

$$(1)\ \ldots\ldots\ldots \frac{\partial C}{\partial S_1} = \frac{1}{S_1}\frac{\partial C}{\partial b}$$

$$\frac{\partial C}{\partial S_2} = \frac{1}{S_2}\frac{\partial C}{\partial d} \ldots\ldots\ldots (2)$$

$$\frac{\partial^2 C}{\partial S_1^2} = \frac{\partial}{\partial S_1}\left(\frac{\partial C}{\partial b}\right)$$

$$= \frac{1}{S_1} = \frac{\partial}{\partial S_1}\left(\frac{\partial C}{\partial b}\right) + \frac{\partial C}{\partial b}\frac{\partial}{\partial S_1}\left(\frac{1}{S_1}\right)$$

$$= \frac{1}{S_1} = \frac{\partial}{\partial b}\left(\frac{\partial c}{\partial b}\right)\frac{\partial b}{\partial S_1} - S_1^2\ \frac{\partial C}{\partial b}$$

$$= \frac{1}{S_1^2}\ \frac{\partial^2 C}{\partial b^2} - \frac{1}{S^2}\ \frac{\partial C}{\partial b}$$

$$(ii) ------ \frac{\partial^2 C}{\partial S_1^2} = \frac{1}{S_1^2} - \frac{\partial^2 C}{\partial b^2} - \frac{\partial C}{\partial b}$$

<u>In the same manner</u>

$$\frac{\partial^2 C}{\partial S_2^2} = \frac{1}{S_2^2}\left[\frac{\partial^2 C}{\partial d^2} - \frac{\partial C}{\partial d}\right] ----- (iii)$$

$$\frac{\partial C}{\partial S_1} = \frac{1}{S_1}\ \frac{\partial C}{\partial b}$$

$$\frac{\partial C}{\partial S_2}\left(\frac{\partial C}{\partial S_1}\right) = \frac{\partial}{\partial S_2}\left(\frac{1}{S_1} - \frac{\partial C}{\partial b}\right)$$

$$= \frac{1}{S_1}\frac{\partial}{\partial S_2}\ \frac{\partial C}{\partial b}$$

$$= \frac{1}{S_1}\frac{\partial}{\partial d}\left(\frac{\partial C}{\partial b}\right)\frac{\partial d}{\partial S_2}$$

$$= \frac{1}{S_1}\frac{\partial^2 C}{\partial b\,\partial d}\ \frac{1}{S^2}$$

$$(iv) ------ \frac{\partial^2 C}{\partial S_1\,\partial S_2}\ \frac{1}{S_2}\ \frac{\partial^2 C}{\partial b\,\partial d}$$

Substitute one, two, three & four in forth equation

$$\left\{\begin{array}{l} \frac{\partial^\propto C}{\partial t^\propto} + \frac{S_1^2\ \sigma_1^2}{\partial s_1^2}\left(\frac{\partial^2 C}{\partial b^2} - \frac{\partial C}{\partial b}\right) + \frac{\sigma^2}{2}\frac{S_2^2}{S_2^2} = \left(\frac{\partial^2 C}{\partial d^2} - \frac{\partial C}{\partial d}\right) + \partial s_1 .\frac{1}{S_1}\ \frac{\partial C}{\partial b} \\ + rS_2 .\frac{1}{S_2}\ \frac{\partial C}{\partial d} + P\ s_1 s_2\ \sigma_1\sigma_2\ \frac{1}{S_1 S_2}\ \sigma_1\sigma_2\ \frac{1}{S_1\ S_2}\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} \frac{\partial^\propto C}{\partial t^\propto} + \frac{\sigma_1^2}{2}\ \frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\ \frac{\partial^2 C}{\partial d^2} - \frac{\sigma_1^2}{2}\frac{\partial p}{\partial b} - \frac{\sigma_2^2}{2}\frac{\partial C}{\partial b} = \left(\frac{\partial^2}{\partial d^2} - \frac{\partial C}{\partial d}\right) \\ + r\frac{\partial C}{\partial b} + r\frac{\partial C}{\partial d} + P\ \sigma_1\sigma_2 - \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0 \end{array}\right.$$

$$\begin{cases} \frac{\partial^{\propto} C}{\partial t^{\propto}} + \frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial d^2} + \left(r\ \frac{\sigma_1^2}{2}\right)\frac{\partial C}{\partial b} + \left(r - \frac{\sigma_2^2}{2}\right)\frac{\partial C}{\partial d} \\ \qquad + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0 \\ \text{Here we need to use substitution method} \end{cases}$$

$$b = lnS_1 - \left(r - \frac{1}{2}\ \sigma_1^2\right)\ o$$

$$d = lnS_2 + \left(r - \frac{1}{2}\ \sqrt{2}^2\right)\ o$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial b}\ \frac{\partial b}{\partial t}$$

$$\boxed{\frac{\partial C}{\partial t} = \frac{\partial C}{\partial b}\left(r - \frac{1}{2}\ \sigma_1^2\right)}$$

$$\frac{-\partial C}{\partial t} = \frac{\partial C}{\partial b}\left(r - \frac{1}{2}\ \sigma_1^2\right)$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial d}\ \frac{\partial d}{\partial t}$$

----------(vii)

$$\boxed{\frac{\partial p}{\partial t} = \frac{\partial C}{\partial d}\left(r - \frac{1}{2}\ \sigma_1^2\right)}$$

From six & seventh equation fifth one becomes.

$$\frac{\partial^{\propto} C}{\partial t^{\propto}} + \frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial d^2} - \frac{\partial C}{\partial t} + \frac{\partial C}{\partial t} + +p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0$$

$$\boxed{\frac{\partial^{\propto} C}{\partial t^{\propto}} + \frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{r_2^2}{2}\frac{\partial^2 C}{\partial d^2} + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0} \text{----------(A)}$$

Subject to:

$$C(b,d,o) = \max\ (w_1\ e^b + w_2\ e^d, 0) \text{ ----------(viii)}$$

Therefore, equation (8) is a simplified European style ; the pricing model of two stock options

Fractional order Black-shole P.D.E

Implement sumudu transform on eight equation

$$\boxed{s\left[\frac{\partial^{\propto} C}{\partial t^{\propto}}\right] = -\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial d^2} + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0}$$

$$b^{-\propto} S\ C - b^{-\propto}(b,d,o) = -s\left[\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2}{\partial d^2}C + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC = 0\right.$$

$$S\ C - C\ (b,d,o) = -b^{\propto} s\left[\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial d^2} + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC\right]$$

$$S\ C = C\ (b,d,o) - b^{\propto} s\left[\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial b^2} + \frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial d^2} + p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\,\partial d} - rC\right. - - - - - \text{(ix)}$$

Implement inverse samudu transform on -----------(ix)

$$C \ = s^{-1}\ C\ (b,d,\infty)\ s^{-1}\ [\ b^{\propto}s\ [\frac{\sigma_1^2}{2}\ \frac{\partial^2 C}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ C}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\ \partial d}-\ rC$$

$$C\ \ (b,d,o)=C\ (b,d,o)\ s^{-1}\ [\ b^{\propto}s\ [\frac{\sigma_1^2}{2}\ \frac{\partial^2 C}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ C}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\ \partial d}-\ rC\ ------(x)$$

with simplification # 14

$$I^{\propto}\ h\ \ (m,n,o)=s^{-1}[b^{\propto}sh\ \ (m,n,o)]$$

Implement Integral property of Sumudu transform on -------------------- (x)

$$C\ \ (b,d,o)=C(b,d,o)\ \ I^{\propto}\ [\frac{\sigma_1^2}{2}\ \frac{\partial^2 C}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ C}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 C}{\partial b\ \partial d}-\ rC]\ -----\ (ix)$$

Sumudu transform showing solution of P.D.E by utilizing equation nine in type of infinite convergent series as mention here.

$$C_0\ \ (b,d,o)=C\ (b,d,o)\ \sum_{j=0}^{\infty}\frac{hj(m,n)t^{j\propto}}{\Gamma(1+j^{\propto})}$$

Where $C\ \ (b,d,o)=h_0\ (b,d)=\ h_0\ (say)$

$$C_{n+1}\ \ (b,d,o)=\ \sum_{j=0}^{\infty}\frac{hj\ o^{j\propto}}{\Gamma(1+j^{\propto})}$$

$$C\ \ (b,d,o)=C\ \ (b,d,o)+\sum_{j=1}^{\infty}\frac{hj(m,n)\ o^{j\propto}}{\Gamma(1+j^{\propto})}$$

be the European call option price solution at time t.

where

$$C\ (b,d,o)=\ h_0$$

$$h_1=-\left[\frac{\sigma_1^2}{2}\ \frac{\partial^2 h_0}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ h_0}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 h_0}{\partial b\ \partial d}-\ rh_0\right]$$

$$h_2=-\left[\frac{\sigma_1^2}{2}\ \frac{\partial^2 h_1}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ h_1}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 h_1}{\partial b\ \partial d}-\ rh_1\right]$$

$$h_3=-\left[\frac{\sigma_1^2}{2}\ \frac{\partial^2 h_2}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ h_2}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 h_2}{\partial b\ \partial d}-\ rh_2\right]$$

$\vdots$

$$h_{n+1}=-\left[\frac{\sigma_1^2}{2}\ \frac{\partial^2 h_n}{\partial b^2}+\frac{\sigma_2^2}{2}\ \frac{\partial^2\ h_n}{\partial d^2}+p\ \sigma_1\sigma_2\ \frac{\partial^2 h_n}{\partial b\ \partial d}-\ rh_n\right]$$

Time Fractional B.S Equation for Two Stocks

Example 1: **(for European Call Option)**

Data:

S_1 = Cost in $ of asset 1.

S_2 = Cost in $ of asset 2.

Table 1.1. Data of cost of asset 1 and 2.

S_1	20	40	70	100	150
S_2	50	80	120	180	200

Condition initially we consider:

C(S_1, S_2, o) =Max($e^{S_1} + 2e^{S_2} - 80, 0$)

The asset 1 practical cost = Rs.80

The asset 2 practical cost= Rs.20

The largest practical cost for = eighty PKR

For european call option.

Time for practical collection = 8 months

$$\propto = 0.005$$

S.D of asset $1 = \sigma_1 \quad = 40\ \%$

S.D of asset $2 = \sigma_2 \quad = 25\ \%$

Section of asset 1 = $w_1 = 2$& Section of asset 2 =w_2=2

Risk free rate of return = 8%

Correlation coefficient between asset 1 and asset 2= 75%

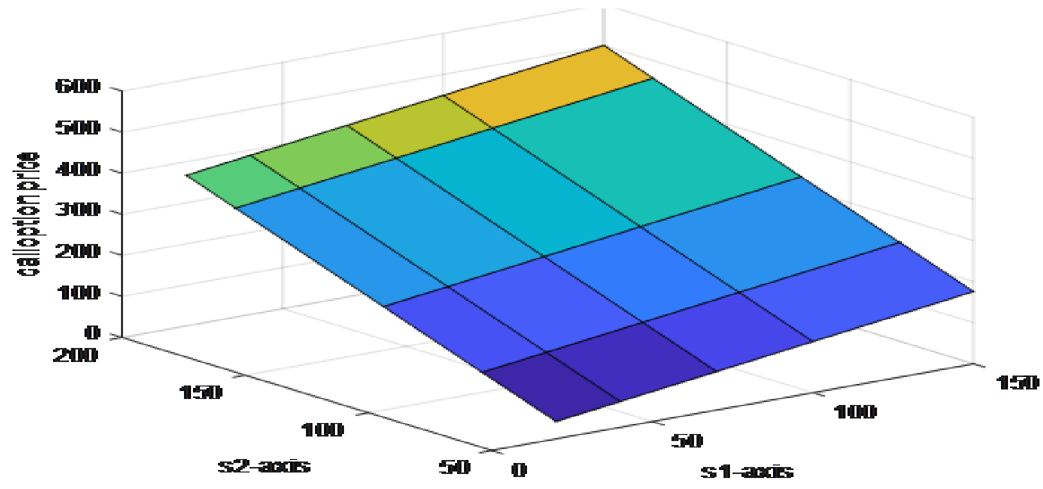
by using Matlab, we solve the above problem then the cost of European call option is mentioned below:

C($S_1, S_2, 0$) = *1.1* exp (S_1) + *2.13* exp (S_2)· - *86.9*

<u>CALL OPTION COSTS</u>

Table 1.2. Call option costs of asset 1 and 2.

		S1				
		20	40	70	100	150
SS2	50	42	62	93	124	175
	80	107	127	158	189	240
	120	193.	214	245	275.9	327.28
	180	323.57	344.12	374.94	405.77	457.15
	200	366.86	387.41	418.23	449.06	500.43



2-D Time fractional ordered european style call option pricing graph

Example 2:

Option type: we need to consider the following data.

S_1 = asset 1 cost in $.

S_2 = asset 2 cost in $

Table 1.3. Data of cost of asset 1 and 2.

S_1	20	40	70	100	150
S_2	50	80	120	180	200

First Condition: $\mathrm{P}(S_1, S_2, 0) = Max((60 - 3\sin\pi s_1 - 5cos\pi s_2), 0)$

asset 1 practical cost = 60

asset 2 practical cost = 20

The largest practical cost = Rs.60

Time for practical collection = Two months

$$\propto = 0.755$$

s.d of asset $1 = \sigma_1 \quad = 45\ \%$

s.d of asset $2 = \sigma_2 \quad = 85\ \%$

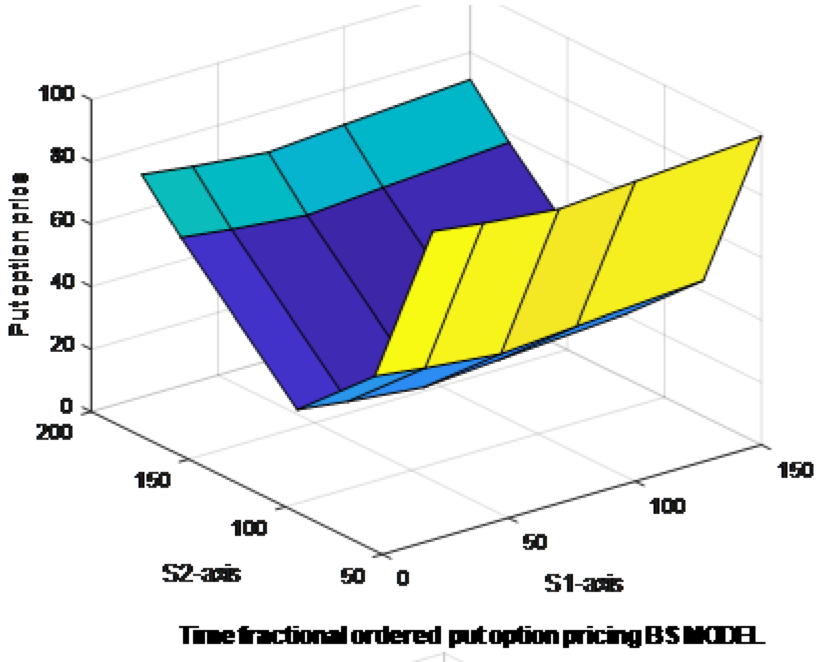
Section of asset 1 = $w_1 = 3$ & section of asset 2 =w_2=5

Risk free rate of return = 03%

asset 1 and asset 2 correlation= 65%

by using Matlab, we solve the above problem then the cost of option is mentioned below:

$\mathbf{P}(S_1, S_2, 0)$ =27.5 cos(3.1 .S_2) - 2.1 cos(3.14. S_1) - 1.5 sin(3.142. S_1) -32.9 sin(3.14. S_2)+ 60.5



Put Option Prices

Table 1.4. Option prices of asset 1 and 2.

		S1				
		20	40	70	100	150
S2	50	98.861	96.764	94.274	96.115	98.926
	80	43.341	41.244	38.754	40.595	43.406
	120	20.404	18.307	15.817	17.658	20.469
	180	57.17	55.072	52.583	54.424	57.235
	200	71.267	69.17	66.68	68.521	71.332

Example 3: (European call option)

Data:

S_1 = Cost of asset 1 in $.

S_2 = Cost of asset 2 in $

Table 1.5. Data of cost of asset 1 and 2.

S_1	20	40	70	100	150
S_2	50	80	120	180	200

I.C: $\text{C}(S_1, S_2, 0) = Max((2S_1^3 + 5S_2^2), 0)$

The practical cost of asset 1 = Sixty

The practical cost of asset 2 = Ninety

The largest practical cost for = Ninety

Time for practical collection = two months

$$\propto = .125$$

s.d of asset $1 = \sigma_1 \quad = 40\ \%$

s.d of asset $2 = \sigma_2 \quad = 65\ \%$

Section of asset 1 = $w_1 = 2$ & section of asset 2 = w_2 = 5

Risk free rate of return = 07%

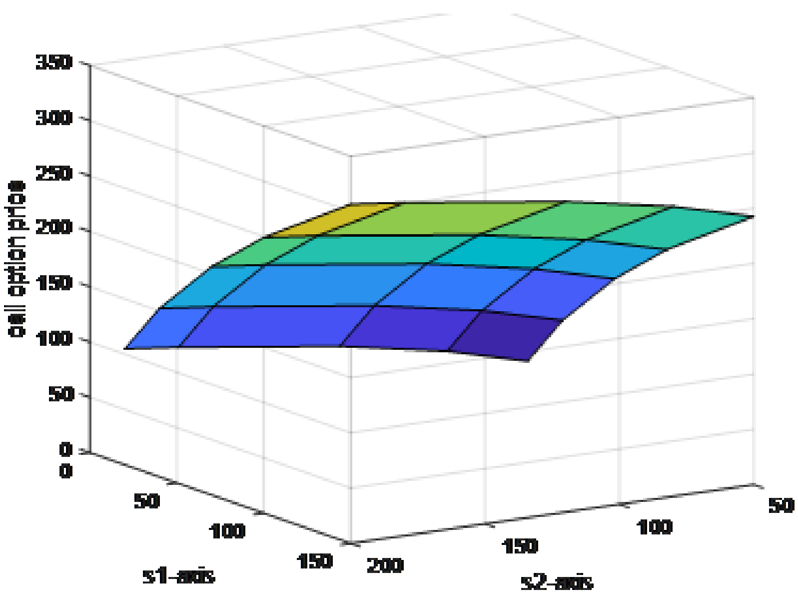
asset 1 and asset 2 correlation= 85%

After solving the above problem then the cost of option is mentioned below:

$$C(S_1, S_2, o) = 2.2\ S_1^3 + 5.4\ \ S_2^2 - 99.8$$

Table 1.6. European Call option of cost of asset 1 and 2.

		S1				
		20	40	70	100	150
S2	50	40.648	88.334	142.39	185.17	242.47
	80	60.194	107.88	161.94	204.71	262.02
	120	78.847	126.53	180.59	223.36	280.67
	180	99.157	146.84	200.9	243.67	300.98
	200	104.71	152.39	206.45	249.22	306.53



2-D Time fractional ordered european style call option pricing

Example 4: (European put option)

Data:

S_1 = Cost of asset 1 in $.

S_2 = Cost of asset 2 in $

Table 1.7. European put option of asset 1 and 2.

S_1	20	40	70	100	150
S_2	50	80	120	180	200

IC: $C(S_1, S_2, o) = Max(2(x^2+y^2)$- ln(y)+ln(x),0)

Largest practical cost for = Rs2.(x^2+y^2)

Time for practical collection = Five months

$$\propto = .125$$

s.d of asset $1 = \sigma_1 \quad = 40$ %

s.d of asset $2 = \sigma_2 \quad = 20$ %

section of asset 1 = $w_1 = 1$& section of asset 2 =w_2=1

Risk free rate of return = 8%

asset 1 and asset 2 correlation**= 75%**

by using Matlab, we solve the above problem then the cost of option is mentioned below:

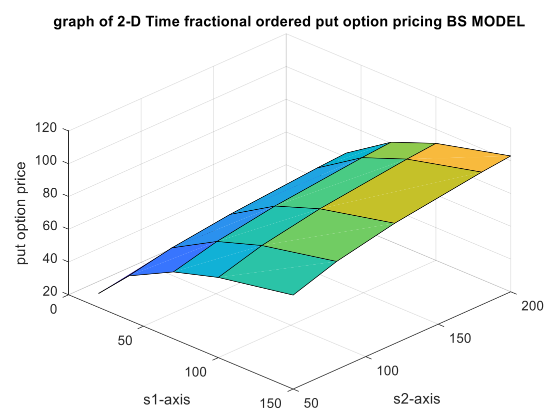
p(S_1, S_2, t)= 2.2 .x^2- 1.1 ln(y) - 1.1 ln(x) + 0.1 /x^3

+0.145 /x^6 + 1.23 /x^9 + 2.2 y^2+ 0.05 /y^2+ 0.1 /y^6 + 0.3 /y^9 - 49

put -option prices

Table 1.8. Put Option of asset 1 and 2.

		S1				
		20	40	70	100	150
S2	50	28.452	46.868	60.826	68.965	77.391
	80	31.964	52.15	67.538	76.588	86.049
	120	34.994	56.708	73.328	83.164	93.519
	180	38.025	61.265	79.119	89.74	100.99
	200	38.812	62.45	80.623	91.449	102.93



Example 5:

Data:

S_1 = Cost of asset 1 in $.

S_2 = Cost of asset 2 in $

Table 1.9. Data of cost of asset 1 and 2.

S_1	20	40	70	100	150
S_2	50	80	120	180	200

IC: $p(S_1, S_2, 0) = Max$(-5 . Sin(x)-8.y+5.x.y, x , 0)

The largest practical cost for = Rs. 25.x.y

Time for practical collection = Five months

$$\propto = .125$$

S.D of asset $1 = \sigma_1 \quad = 40$ %

S.D of asset $2 = \sigma_2 \quad = 20$ %

Section of asset 1 = $w_1 = 1$& section of asset 2 =w_2=1

Risk-free-rate = 8%

asset 1 and asset 2 correlation= **75%**

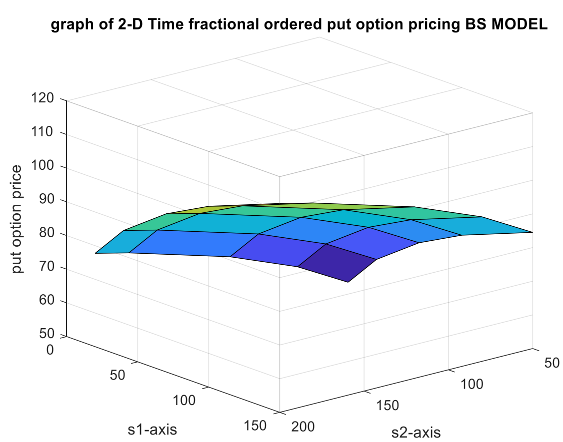
by using Matlab, we solve the above problem then the cost of option is mentioned below:

$P(S_1, S_2, t)$= 5.4 .x.y - 0.18.cos(x) - 5.4 .sin(x) - 8.6 y - 0.221

put-option costs

Table 1.10. Put option costs of asset 1 and 2.

		S1				
		20	40	70	100	150
S2	50	50.18	59.961	69.4	76.156	84.502
	80	58.571	68.353	77.8	84.548	92.894
	120	66.592	76.374	85.8	92.568	100.91
	180	75.335	85.117	94.5	101.31	109.66
	200	77.725	87.507	96.9	103.7	112.05



CONCLUSION

According to the above discussion and graphical examples, it is concluded that the model of BS PDE can be considered as a solid tool for prediction of option pricing. The use of the Samudu Transform has decreased the handling time of the equations. The defined instances also appeared as the effortlessness, productivity, and reliability of the proposed strategy. This demonstrates that Samudu-Transformation provides a simple and time-effective solution for the e-commerce industry.

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